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MAGNETOSTATIC-WAVE PROPAGATION IN A FINITE YIG-LOADED RECTANGULAR WAVEGUIDE*

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Abstract

The propagation of magnetostatic waves (MSW) in a waveguide partially loaded with a low-loss YIG slab is investigated theoretically. Using the integral equation method, the dispersion relation is found to be an infinitely large determinant equal to zero. Proper truncation of this determinant and numerical analysis to find its roots are carried out in this work. It is noticed that there exists a trade off between the time delay and the device bandwidth and maximization of one property leads to a poor value in the other. Thus some design compromises should be made. It is also observed that the frequency range of operation of the device can be adjusted by an external magnetic bias field. This flexibility in tuning the device to operate in any frequency range adds an extra dimension of flexibility to the operation and also design of these devices.

Introduction

Magnetostatic-wave propagation in a YIG slab in free space on an infinite ground plane or bounded by two infinite parallel ground planes or completely filling a waveguide has been reported in the literature.[1-3] Some results pertinent to the design and construction of delay lines and filters were also given.[4-6] The theoretical analysis carried out by all these previous works are based on the method of separation of variables, whereby a closed form for the dispersion relation may be obtained.

In this paper, the propagation of magnetostatic waves in a rectangular waveguide partially filled with a YIG slab is studied theoretically. The dc external magnetic field is parallel to the slab and perpendicular to the direction of propagation. The slab is placed inside and along the guide but not necessarily in contact with the waveguide walls. Figure 1 shows a cross section of the configuration. To simplify the analysis, the slab is assumed to be thin, so that approximate numerical solution becomes feasible.

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The introduction of a gap length (x_0) is motivated to account for the loose contacts between the YIG slab and the waveguide walls and to provide a general structure for the design of delay lines. For the configuration shown in Figure 1, if the gap length x_0 is zero, conventional mode analysis may be used to solve for the dispersion relation.[7] However, when x_0 is nonzero, numerical analysis based on the integral equation formulation appears to be the only means.

Based on the integral equation method the problem of magnetostatic-wave propagation in a YIG slab of finite width inside a waveguide (see Fig 1) is analyzed and the magnetic potential function in the YIG region is expressed in terms of an integral equation. Assuming the slab to be thin, an approximate numerical solution to the integral equation can be obtained. Based on the approximate numerical solution for the dispersion relation, some computer results obtained in a certain frequency range is plotted. The results obtained from the approximate numerical solution for the general case, when it is reduced to special cases, is in good agreement with the existing published results.

Theoretical Analysis

As noted earlier, when the width of the slab is less than the width of the waveguide, that is when $x_0 \neq 0$ (see Fig 1), the mode analysis technique appears to be fruitless and the integral equation method seems to be more appropriate. In this method an unknown magnetic potential function denoted by $\Phi(x,y,z)$ at a point (x,y,z) inside the YIG slab is assumed to exist. Assuming a time dependence of the form $e^{j\omega t}$ and wave propagation in the y-direction, the y-variation would be therefore of the form e^{-jky} where ω, t and k are the frequency of operation, the time parameter, and the wave number respectively. In this manner the magnetic potential function in the YIG region, $\Phi(x,y,z)$, can be written as $\tilde{\Phi}(x,z)e^{-jky}$. Based on $\tilde{\Phi}(x,z)$ and with the help of proper permeability tensor,[8] fictitious "magnetic sources" can be obtained in terms of $\tilde{\Phi}(x,z)$. The density of magnetic sources consists of two parts a) the magnetic volume charge density (ρ_V) and b) the magnetic surface charge density (ρ_S). Considering a uniform guide cross section and combining the obtained magnetic sources with an appropriate Green's function, an integral expression for the magnetic potential function $\tilde{\Phi}(x,z)$ in the YIG region (except for the common factor e^{-jky}) can be written as:

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$$\tilde{\Phi}(x,z) = \iint\limits_{\substack{\text{YIG}\\ \text{area}}} \rho_V(x', z')G(x, x', z, z')dx'dz' + \int\limits_{\substack{\text{YIG}\\ \text{sides}}} \rho_S(x', z')G(x, x', z, z')dz' . \quad (1)$$

where $G(x,x',z,z')$ is the Green's function for a magnetic line source located at (x',z') inside the waveguide.

Approximate Numerical Solution

The two dimensional integral expression given by equation (1) is difficult to solve numerically. Assuming the slab to be very thin makes this equation one dimensional and tractable. This assumption also makes the demagnetizing forces negligible in this analysis.[9]

With this assumption and some further mathematical manipulation, an integro-differential equation in terms of $\tilde{\Phi}(x,z)$ is obtained. Solving the equation so obtained for the dispersion relations would lead to an infinite system of linear equations which are coupled. Decoupling this system of equations into two sub-systems, one for odd modes and another for even modes, would lead to two matrix equations. Each matrix equation consists of an infinite matrix multiplied by a constant vector.

For nontrivial unique solutions to the matrix equations, the infinite determinant of the coefficient matrix must be zero in theory. The dispersion relation is therefore obtained from the vanishing of this infinite determinant. However, in practice the size of the matrix is reduced by finding a proper cutoff point (N). Denoting the determinant of the cutoff (NxN) matrix by $D_N(\omega,k)$, the equation to be solved can be written as:

$$D_N(\omega,k) = 0 \quad (2)$$

Root finding of the dispersion relation as given by equation (2) is done by the Newton-Raphson method. With the aid of a proper algorithm and computer programming, the determinant roots of the dispersion relation were found through several iterations.

Computer Simulation and Results

Considering only the first mode and with the aid of a computer program, several important effects were studied. For viewing purposes, the results of the root finding procedures were plotted. Figure 2 shows the relationship between the normalized air-gap $(2x_o/a)$ and the cutoff point N. As it can be noticed from this figure, there is roughly an exponential increase in matrix size as the normalized air-gap increases.

Figure 3 shows the effect of increasing the normalized air-gap $(2x_o/a)$ on the dispersion relation. It is noted that the dispersion curves shift downward as the air-gap increases. The combined effects of the position and width of the YIG slab is shown in Figure 4. In this figure, the dispersion relations for several positions (z_o) of the slab, each position with two values of x_o, are presented. From this figure it is noticed that as the slab position is lowered, the dispersion curves are compressed with smaller bandwidths. The corresponding group time delay per unit length in ns/cm defined by the relation:

$$\tau_d = \left(\frac{\partial\omega}{\partial k}\right)^{-1} \quad (3)$$

is shown in Figure 5. In this figure, for fixed time delay the operating frequency can be adjusted effectively by varying the position z_o, while for a fixed frequency the time delay can be increased by increasing the gap length x_o.

Tunable properties are also investigated by varying the magnetic bias field. Figure 6 shows the effect of magnetic bias field on the dispersion curves for a special case, that is when $x_o = 0$. As can be seen, the dispersion curves move up or down the ω-k plane by increasing or decreasing H_{dc} respectively.

Conclusions

The propagation and time delay characteristics of magnetostatic waves in a waveguide partially filled with a YIG slab, with an equal air-gap on each of its sides, were studied and some numerical results were presented for several chosen configurations over a frequency range of approximately 6.0 to 10.0 GHZ. The dependence of the dispersion relation and group time delay per unit length on the position and width of the YIG slab were presented. The effects of varying H_{dc} on the dispersion curves were also studied and presented.

It is concluded that as the slab width decreases, the delay time increases and dispersion curves bandwidth shifts downward while as the slab position is lowered, the delay time increases and the dispersion curves are compressed with smaller bandwidths. This means that roughtly speaking, the position of the slab controls the bandwidth and its width controls the center frequency of the device.

From Figures 4 and 5, it is seen that in order to obtain high values of group time delay per unit length, the YIG slab must be narrow and placed at the bottom of the guide. On the other hand, to maximize the device bandwidth, a narrow YIG slab positioned at the top inside surface of the waveguide is preferred. Therefore, it can be concluded that there exists a trade off between the time delay per unit length and the device bandwidth and maximization of one property leads to a poor value in the other. Thus some design compromises should be made.

Finally, the ability of the device to be tuned by means of a magnetic bias field adds an extra dimension of flexibility for its operation in any desired frequency range.

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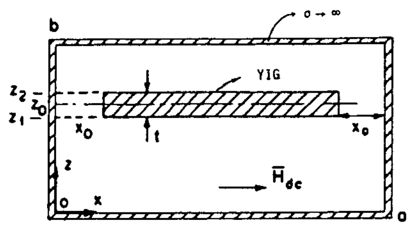

FIG. 1 DEVICE CONFIGURATION.

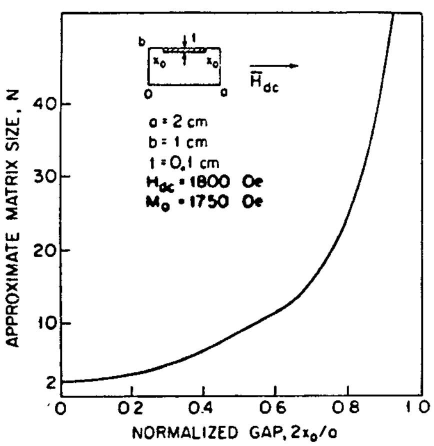

FIG. 2 RELATIONSHIP OF THE WALL GAP (x_0) AND THE CUT-OFF POINT (N).

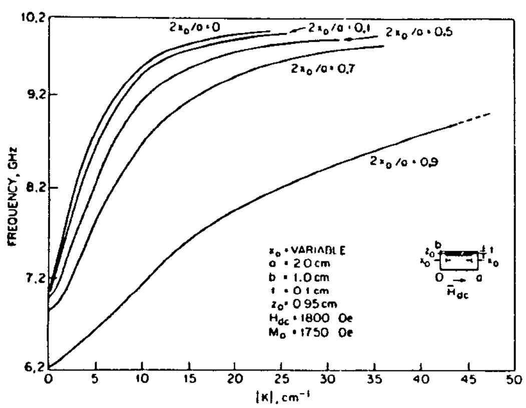

FIG. 3 EFFECT OF INCREASING THE AIR GAP (x_0) ON THE DISPERSION CURVE.

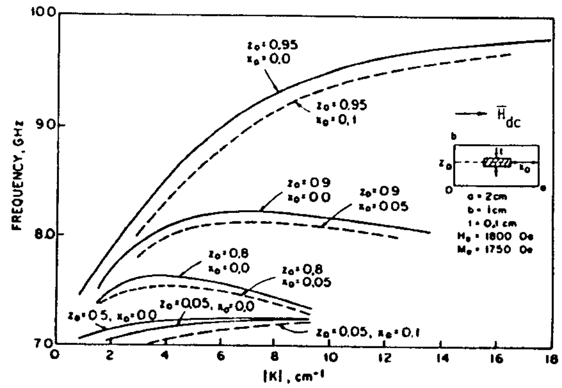

FIG. 4 COMBINED EFFECT OF POSITION AND WIDTH OF THE SLAB ON THE DISPERSION CURVES.

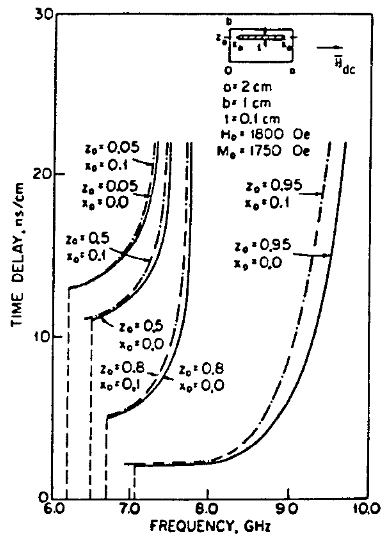

FIG 5 EFFECT OF SLAB WIDTH AND POSITION ON TIME DELAY/UNIT LENGTH.

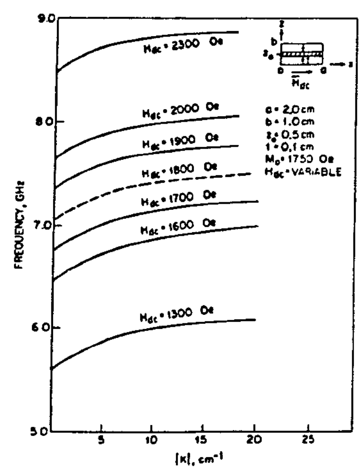

FIG 6 EFFECT OF MAGNETIC BIAS FIELD